From: PRECALCULUS by Michael Sullivan

Arithmetic of Quotients

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \quad b \neq 0, d \neq 0$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad b \neq 0, d \neq 0$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}, \quad b \neq 0, c \neq 0, d \neq 0$$

Special Products/Factoring Formulas

$(x + a)(x - a) = x^2 - a^2$

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x - a)(x^2 + ax + a^2) = x^3 - a^3$

$(x + a)(x^2 - ax + a^2) = x^3 + a^3$

$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$

$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

Laws of Exponents

$a^m a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$

Properties of Radicals

$\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{m/n}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Properties of Logarithms

$\log_a MN = \log_a M + \log_a N$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$\log_a M^r = r \log_a M$

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

Properties of Inequalities

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Complex Numbers

$i^2 = -1$

$(a + bi) + (c + di) = (a + c) + (b + d)i$

$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

$\overline{a + bi} = a - bi$

Permutations/Combinations

$0! = 1 \qquad 1! = 1$

$n! = n(n - 1) \cdot \cdots \cdot (3)(2)(1)$

$$P(n, r) = \frac{n!}{(n - r)!}$$

$$C(n, r) = \binom{n}{r} = \frac{n!}{(n - r)!r!}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}ba^{n-1} + \binom{n}{2}b^2a^{n-2} + \cdots + \binom{n}{1}b^{n-1}a + b^n$$

Arithmetic Sequence

$$a + (a + d) + (a + 2d) + \cdots + [a + (n - 1)d] = na + \frac{n(n - 1)}{2}d$$

Geometric Sequence

$$a + ar + ar^2 + \cdots + ar^{n-1} = a\frac{1 - r^n}{1 - r}$$

Geometric Series

$$\text{If } |r| < 1,\ a + ar + ar^2 + \cdots = \sum_{k=1}^{\infty} ar^k = \frac{a}{1 - r}$$

Formulas/Equations

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, the distance from P_1 to P_2 is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation of a Circle

The equation of a circle of radius r with center at (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2$$

Slope of a Line

The slope m of the line containing the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{if } x_1 \neq x_2$$

$$m \text{ is undefined} \qquad \text{if } x_1 = x_2$$

Point-Slope Equation of a Line

The equation of a line with slope m containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Slope-Intercept Equation of a Line

The equation of a line with slope m and y-intercept b is

$$y = mx + b$$

Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$, $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$, there are two real unequal solutions.
If $b^2 - 4ac = 0$, there is a repeated real solution.
If $b^2 - 4ac < 0$, there are two complex solutions.

Functions

Constant Function $f(x) = b$

Linear Function $f(x) = mx + b$, m is slope, b is y-intercept

Quadratic Function $f(x) = ax^2 + bx + c$

Polynomial Function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdot \cdot \cdot + a_1 x + a_0$

Rational Function

$$R(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdot \cdot \cdot + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdot \cdot \cdot + b_1 x + b_0}$$

Exponential Function $f(x) = a^x$, $a > 0$, $a \neq 1$

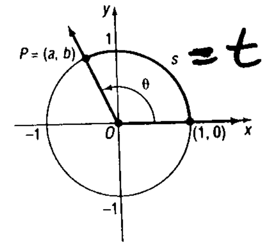
Logarithmic Function $f(x) = \log_a x$, $a > 0$, $a \neq 1$

Trigonometric Functions

Of a Real Number

$\sin t = b$ $\qquad \cos t = a$ $\qquad \tan t = \dfrac{b}{a},\ a \neq 0$

$\csc t = \dfrac{1}{b},\ b \neq 0$ $\qquad \sec t = \dfrac{1}{a},\ a \neq 0$ $\qquad \cot t = \dfrac{a}{b},\ b \neq 0$

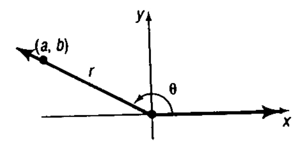


Unit circle, $x^2 + y^2 = 1$
$\theta = t$ radians; $s = t$ units

Of a General Angle

$\sin \theta = \dfrac{b}{r}$ $\qquad \cos \theta = \dfrac{a}{r}$ $\qquad \tan \theta = \dfrac{b}{a},\ a \neq 0$

$\csc \theta = \dfrac{r}{b},\ b \neq 0$ $\qquad \sec \theta = \dfrac{r}{a},\ a \neq 0$ $\qquad \cot \theta = \dfrac{a}{b},\ b \neq 0$



Trigonometric Identities

Fundamental Identities

$\tan \theta = \dfrac{\sin \theta}{\cos \theta}$ $\qquad \cot \theta = \dfrac{\cos \theta}{\sin \theta}$

$\csc \theta = \dfrac{1}{\sin \theta}$ $\qquad \sec \theta = \dfrac{1}{\cos \theta}$ $\qquad \cot \theta = \dfrac{1}{\tan \theta}$

$\sin^2 \theta + \cos^2 \theta = 1$

$\tan^2 \theta + 1 = \sec^2 \theta$

$1 + \cot^2 \theta = \csc^2 \theta$

Even-Odd Identities

$\sin(-\theta) = -\sin \theta$ $\qquad \csc(-\theta) = -\csc \theta$

$\cos(-\theta) = \cos \theta$ $\qquad \sec(-\theta) = \sec \theta$

$\tan(-\theta) = -\tan \theta$ $\qquad \cot(-\theta) = -\cot \theta$

Sum and Difference Formulas

$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Half-Angle Formulas

$$\sin \frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

Double-Angle Formulas

$\sin 2\theta = 2 \sin \theta \cos \theta$

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$\cos 2\theta = 2\cos^2 \theta - 1$

$\cos 2\theta = 1 - 2\sin^2 \theta$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Product-to-Sum Formulas

$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

Sum-to-Product Formulas

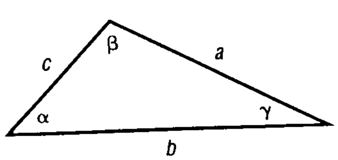
$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

Solving Triangles



Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

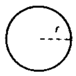
$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

Geometry Formulas

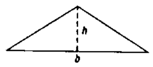
Circle



r = Radius, A = Area, C = Circumference

$A = \pi r^2 \qquad C = 2\pi r$

Triangle



b = Base, h = Altitude (Height), A = area

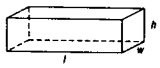
$A = \frac{1}{2}bh$

Rectangle

l = Length, w = Width, A = area, P = perimeter

$A = lw \qquad P = 2l + 2w$

Rectangular Box



l = Length, w = Width, h = Height, V = Volume

$V = lwh$

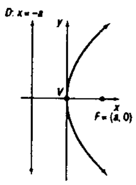
Sphere

r = Radius, V = Volume, S = Surface area

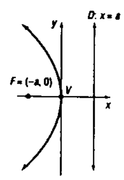
$V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$

Conics

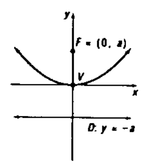
Parabola



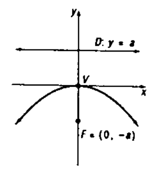
$y^2 = 4ax$



$y^2 = -4ax$

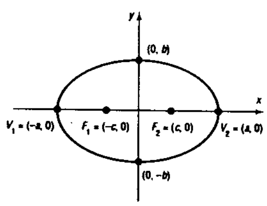


$x^2 = 4ay$

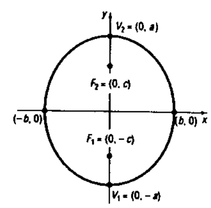


$x^2 = -4ay$

Ellipse

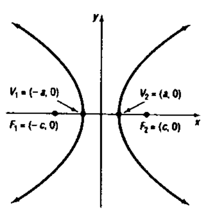


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad c^2 = a^2 - b^2$$



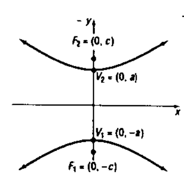
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad c^2 = a^2 - b^2$$

Hyperbola



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad c^2 = a^2 + b^2$$

Asymptotes: $y = \frac{b}{a}x, \quad y = -\frac{b}{a}x$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1, \quad c^2 = a^2 + b^2$$

Asymptotes: $y = \frac{a}{b}x, \quad y = -\frac{a}{b}x$